

# NAG C Library Function Document

## nag\_zggbal (f08wvc)

### 1 Purpose

nag\_zggbal (f08wvc) balances a pair of complex square matrices  $(A, B)$  of order  $n$ . Balancing usually improves the accuracy of computed generalized eigenvalues and eigenvectors.

### 2 Specification

```
void nag_zggbal (Nag_OrderType order, Nag_JobType job, Integer n, Complex a[],
                Integer pda, Complex b[], Integer pdb, Integer *ilo, Integer *ihi,
                double lscale[], double rscale[], NagError *fail)
```

### 3 Description

Balancing may reduce the 1-norm of the matrices and improve the accuracy of the computed eigenvalues and eigenvectors in the complex generalized eigenvalue problem

$$Ax = \lambda Bx.$$

nag\_zggbal (f08wvc) is usually the first step in the solution of the above generalized eigenvalue problem. Balancing is optional but it is highly recommended.

The term ‘balancing’ covers two steps, each of which involves similarity transformations on  $A$  and  $B$ . The function can perform either or both of these steps. Both steps are optional.

1. The function first attempts to permute  $A$  and  $B$  to block upper triangular form by a similarity transformation:

$$PAP^T = F = \begin{pmatrix} F_{11} & F_{12} & F_{13} \\ & F_{22} & F_{23} \\ & & F_{33} \end{pmatrix}$$

$$PBP^T = G = \begin{pmatrix} G_{11} & G_{12} & G_{13} \\ & G_{22} & G_{23} \\ & & G_{33} \end{pmatrix}$$

where  $P$  is a permutation matrix,  $F_{11}$ ,  $F_{33}$ ,  $G_{11}$  and  $G_{33}$  are upper triangular. Then the diagonal elements of the matrix pairs  $(F_{11}, G_{11})$  and  $(F_{33}, G_{33})$  are generalized eigenvalues of  $(A, B)$ . The rest of the generalized eigenvalues are given by the matrix pair  $(F_{22}, G_{22})$  which are in rows and columns  $i_{lo}$  to  $i_{hi}$ . Subsequent operations to compute the generalized eigenvalues of  $(A, B)$  need only be applied to the matrix pair  $(F_{22}, G_{22})$ ; this can save a significant amount of work if  $i_{lo} > 1$  and  $i_{hi} < n$ . If no suitable permutation exists (as is often the case), the function sets  $i_{lo} = 1$  and  $i_{hi} = n$ .

2. The function applies a diagonal similarity transformation to  $(F, G)$ , to make the rows and columns of  $(F_{22}, G_{22})$  as close in norm as possible:

$$DF\hat{D} = \begin{pmatrix} I & 0 & 0 \\ 0 & D_{22} & 0 \\ 0 & 0 & I \end{pmatrix} \begin{pmatrix} F_{11} & F_{12} & F_{13} \\ & F_{22} & F_{23} \\ & & F_{33} \end{pmatrix} \begin{pmatrix} I & 0 & 0 \\ 0 & \hat{D}_{22} & 0 \\ 0 & 0 & I \end{pmatrix}$$

$$DGD^{-1} = \begin{pmatrix} I & 0 & 0 \\ 0 & D_{22} & 0 \\ 0 & 0 & I \end{pmatrix} \begin{pmatrix} G_{11} & G_{12} & G_{13} \\ & G_{22} & G_{23} \\ & & G_{33} \end{pmatrix} \begin{pmatrix} I & 0 & 0 \\ 0 & \hat{D}_{22} & 0 \\ 0 & 0 & I \end{pmatrix}$$

This transformation usually improves the accuracy of computed generalized eigenvalues and eigenvectors.

## 4 References

Ward R C (1981) Balancing the generalized eigenvalue problem *SIAM J. Sci. Stat. Comp.* **2** 141–152

## 5 Parameters

- 1: **order** – Nag\_OrderType *Input*  
*On entry:* the **order** parameter specifies the two-dimensional storage scheme being used, i.e., row-major ordering or column-major ordering. C language defined storage is specified by **order** = **Nag\_RowMajor**. See Section 2.2.1.4 of the Essential Introduction for a more detailed explanation of the use of this parameter.  
*Constraint:* **order** = **Nag\_RowMajor** or **Nag\_ColMajor**.
- 2: **job** – Nag\_JobType *Input*  
*On entry:* specifies the operations to be performed on matrices *A* and *B*:  
 if **job** = **Nag\_DoNothing**, no balancing is done. Initialize **ilo** = 1, **ihi** = **n**, **lscale**[*i* – 1] = 1.0 and **rscale**[*i* – 1] = 1.0, for *i* = 1, ..., *n*;  
 if **job** = **Nag\_Permute**, only permutations are used in balancing;  
 if **job** = **Nag\_Scale**, only scalings are used in balancing;  
 if **job** = **Nag\_DoBoth**, both permutations and scalings are used in balancing.  
*Constraint:* **job** = **Nag\_DoNothing**, **Nag\_Permute**, **Nag\_Scale** or **Nag\_DoBoth**.
- 3: **n** – Integer *Input*  
*On entry:* *n*, the order of the matrices *A* and *B*.  
*Constraint:* **n** ≥ 0.
- 4: **a**[*dim*] – Complex *Input/Output*  
**Note:** the dimension, *dim*, of the array **a** must be at least max(1, **pda** × **n**).  
 Where **A**(*i*, *j*) appears in this document, it refers to the array element  
 if **order** = **Nag\_ColMajor**, **a**[(*j* – 1) × **pda** + *i* – 1];  
 if **order** = **Nag\_RowMajor**, **a**[(*i* – 1) × **pda** + *j* – 1].  
*On entry:* the *n* by *n* matrix *A*.  
*On exit:* **a** is overwritten by the balanced matrix.  
**a** is not referenced if **job** = **Nag\_DoNothing**.
- 5: **pda** – Integer *Input*  
*On entry:* the stride separating matrix row or column elements (depending on the value of **order**) in the array **a**.  
*Constraint:* **pda** ≥ max(1, **n**).
- 6: **b**[*dim*] – Complex *Input/Output*  
**Note:** the dimension, *dim*, of the array **b** must be at least max(1, **pdb** × **n**).  
 Where **B**(*i*, *j*) appears in this document, it refers to the array element  
 if **order** = **Nag\_ColMajor**, **b**[(*j* – 1) × **pdb** + *i* – 1];  
 if **order** = **Nag\_RowMajor**, **b**[(*i* – 1) × **pdb** + *j* – 1].  
*On entry:* the *n* by *n* matrix *B*.

On exit: **b** is overwritten by the balanced matrix.

**b** is not referenced if **job** = **Nag\_DoNothing**.

7: **pdb** – Integer Input

On entry: the stride separating matrix row or column elements (depending on the value of **order**) in the array **b**.

Constraint: **pdb**  $\geq \max(1, \mathbf{n})$ .

8: **ilo** – Integer \* Output

9: **ihi** – Integer \* Output

On exit:  $i_{lo}$  and  $i_{hi}$  are set such that  $\mathbf{A}(i, j) = 0$  and  $\mathbf{B}(i, j) = 0$  if  $i > j$  and  $1 \leq j < i_{lo}$  or  $i_{hi} < i \leq n$ .

If **job** = **Nag\_DoNothing** or **Nag\_Scale**,  $i_{lo} = 1$  and  $i_{hi} = n$ .

10: **lscale**[*dim*] – double Output

**Note:** the dimension, *dim*, of the array **lscale** must be at least  $\max(1, \mathbf{n})$ .

On exit: details of the permutations and scaling factors applied to the left side of the matrices *A* and *B*. If  $P_i$  is the index of the row interchanged with row  $i$  and  $d_i$  is the scaling factor applied to row  $i$ , then

**lscale**[ $i - 1$ ] =  $P_i$ , for  $i = 1, \dots, i_{lo} - 1$ ;

**lscale**[ $i - 1$ ] =  $d_i$ , for  $i = i_{lo}, \dots, i_{hi}$ ;

**lscale**[ $i - 1$ ] =  $P_i$ , for  $i = i_{hi} + 1, \dots, n$ .

The order in which the interchanges are made is  $n$  to  $i_{hi} + 1$ , then 1 to  $i_{lo} - 1$ .

11: **rscale**[*dim*] – double Output

**Note:** the dimension, *dim*, of the array **rscale** must be at least  $\max(1, \mathbf{n})$ .

On exit: details of the permutations and scaling factors applied to the right side of the matrices *A* and *B*. If  $P_j$  is the index of the column interchanged with column  $j$  and  $\hat{d}_j$  is the scaling factor applied to column  $j$ , then

**rscale**[ $j - 1$ ] =  $P_j$ , for  $j = 1, \dots, i_{lo} - 1$ ;

**rscale**[ $j - 1$ ] =  $\hat{d}_j$ , for  $j = i_{lo}, \dots, i_{hi}$ ;

**rscale**[ $j - 1$ ] =  $P_j$ , for  $j = i_{hi} + 1, \dots, n$ .

The order in which the interchanges are made is  $n$  to  $i_{hi} + 1$ , then 1 to  $i_{lo} - 1$ .

12: **fail** – NagError \* Output

The NAG error parameter (see the Essential Introduction).

## 6 Error Indicators and Warnings

### NE\_INT

On entry, **n** =  $\langle \text{value} \rangle$ .

Constraint: **n**  $\geq 0$ .

On entry, **pda** =  $\langle \text{value} \rangle$ .

Constraint: **pda**  $> 0$ .

On entry, **pdb** =  $\langle \text{value} \rangle$ .

Constraint: **pdb**  $> 0$ .

**NE\_INT\_2**

On entry, **pda** =  $\langle value \rangle$ , **n** =  $\langle value \rangle$ .  
 Constraint: **pda**  $\geq$  max(1, **n**).

On entry, **pdb** =  $\langle value \rangle$ , **n** =  $\langle value \rangle$ .  
 Constraint: **pdb**  $\geq$  max(1, **n**).

**NE\_ALLOC\_FAIL**

Memory allocation failed.

**NE\_BAD\_PARAM**

On entry, parameter  $\langle value \rangle$  had an illegal value.

**NE\_INTERNAL\_ERROR**

An internal error has occurred in this function. Check the function call and any array sizes. If the call is correct then please consult NAG for assistance.

**7 Accuracy**

The errors are negligible, compared to those in subsequent computations.

**8 Further Comments**

nag\_zggbal (f08wvc) is usually the first step in computing the complex generalized eigenvalue problem but it is an optional step. The matrix  $B$  is reduced to the triangular form using the  $QR$  factorization function nag\_zgeqrf (f08asc) and the unitary transformation  $Q$  is applied to the matrix  $A$  by calling nag\_zunmqr (f08auc). This is followed by nag\_zgghrd (f08wsc) which reduces the matrix pair into the generalized Hessenberg form.

If the matrix pair  $(A, B)$  is balanced by this function, then any generalized eigenvectors computed subsequently are eigenvectors of the balanced matrix pair. In that case, to compute the generalized eigenvectors of the original matrix, nag\_zggbak (f08wwc) must be called.

The total number of floating-point operations is approximately proportional to  $n^2$ .

The real analogue of this function is nag\_dggbal (f08whc).

**9 Example**

See Section 9 of the documents for nag\_zhgeqz (f08xsc) and nag\_ztgevc (f08yxc).

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